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REVIEWS

Weak Chaos and Quasi-Regular Patterns. By G. M. Zaslavsky, R. Z. Sagdeev, D. A. Usikov and A. A. Chernikov. Cambridge University Press, 1991. 253 pp. £40 or $75.

This monograph, first in the Cambridge Nonlinear Science series, deals with the occurrence of chaotic behaviour in almost integrable Hamiltonian systems. In such systems, chaotic behaviour is confined to small areas of phase space associated with the separatrices of the unperturbed integrable system; these structures have come to be known as 'webs' on account of their appearance. When the perturbations obey certain resonances with respect to the unperturbed system, the webs can take on interesting symmetrical or 'quasi-crystalline' forms (for example, with 5-fold symmetry). After a general introduction and analysis of detailed models of web formation and bifurcation, there is a description of the analogy with chaotic streamlines in time-dependent 2-D and 3-D hydrodynamics. Later there is a more speculative section on tilings and patterns in nature.

The introductory matter (chapters 1–6) is in general admirable, giving in a short space the important elements of KAM theory and Arnold diffusion in language that is not too formal. The development by means of worked examples provides clarity, while the reputation of the authors is available to convince us that we are learning about general rather than special properties of the underlying systems. While the selection of diagrams and graphs is good, the quality of some of the illustrations of stochastic regions is poor in relation to modern graphical capabilities. This is more of an editorial problem than a criticism of the authors, and is part of a more general editorial insufficiency which I discuss below. Chapter 7 gives a disquisition on patterns in general, and their description in terms of Hamiltonians with appropriate symmetry. An attempt is made to link this theory with the preceding development for the webs, but this does not seem to lead very far; the symmetries that are responsible for the morphology of the patterns also control the structure of the web, but there is no direct *dynamical* analogy.

Chapters 8 and 9 attempt to relate patterns in hydrodynamics to the earlier theory. This is for me the least satisfactory part of the enterprise. The authors give only the most superficial idea of the mechanisms of pattern formation in (for example) thermal convection in large systems. At the beginning of chapter 8 they imply, perhaps unintentionally, that they believe the formation of convective patterns to be intimately connected with the physics of web formation. No mention is made of the vast literature on bifurcations with symmetry; most stability theorists would agree that the latter work provides a convincing explanation of the occurrence and interaction of patterns in weakly nonlinear situations. The main thrust of this chapter is the investigation of the forms of Kolmogorov-type flows with various symmetries of the forcing function. Intricate quasi-crystalline patterns can result when the forcings are specially chosen. However, here it would seem that the desire to exhibit attractive patterns that resemble the structures arising in the Hamiltonian systems has overcome the need to consider realistic flows. In chapter 9 three-dimensional flows with chaotic particle paths are discussed. Here there is a clear connection with the Hamiltonian material presented earlier, and indeed the relationship between the occurrence of chaotic regions near separatrices of cellular patterns under the action of perturbations is very illuminating. However, there is no discussion of dynamics in this chapter; many of the flows with simple symmetries that are exhibited are almost certainly unstable. Finally in chapter 10 science is abandoned completely for an essay on patterns in art and nature, containing many remarkable illustrations of, for example, Moorish tilings, Escher graphics and pictures of echinoderms from the Haeckel atlas. No attempt is made to relate these patterns to the theory of previous chapters.

In summary, then, this book is an amalgam of disparate elements, not all handled with the same success. The impression is of experts who have, because of their fascination with the appearance of patterns with 5- and higher-fold symmetry in perturbed Hamiltonian systems, been tempted to make much of rather tenuous connections with patterns arising in other contexts. The early chapters, however, are in my opinion interesting enough to make the overall effort very worthwhile. My only serious criticism is reserved for the publishers and editors of the series, who have done a disservice to the authors by permitting the English to remain idiosyncratic rather than idiomatic. In spite of the heroic efforts of the translator, there are several unusual usages; though the meaning is generally clear these infelicities often get in the way of a proper appreciation of the text. Had the latter been a paper in this Journal, many of the problems would have been ironed out by good copy-editing; one would hope to see the same standards applied to monographs too.

MICHAEL PROCTOR

Perturbation Methods. By E. J. HINCH. Cambridge University Press, 1991. 160 pp. £12.95.

It is almost thirty years since Van Dyke's stimulating book *Perturbation Methods in Fluid Mechanics* appeared, with its introduction to singular perturbations and how to deal with them. Techniques have developed since then, further books have appeared and these methods now appear in a reasonable proportion of new books on mathematical methods and differential equations. Readers of this journal will be interested to see how an active and well-known fluid dynamicist now approaches the topic.

As may be expected, by far the most substantial chapter, of 49 pages, is on matched asymptotic expansions. Its structure is similar to that of other chapters, with successively more difficult problems being treated. These go well beyond an example where Van Dyke's matching rule fails, up to 'a terrible problem' which is 'unusually difficult' in that even an intermediate-variable method of matching 'struggles'. The final examples in the chapter are from a range of applications in fluid mechanics and mechanics.

The preceding chapters introduce basic ideas in the context of algebraic equations before a brief chapter on the basics of asymptotic expansions. Then integrals are treated, mainly with Watson's Lemma and the method of steepest descents. Regular perturbations are given a brief (five-page) chapter.

The only other substantial chapter in the book is on the method of multiple scales. After a number of initial examples it introduces WKBJ, turning points, ray theory and the averaged Lagrangian in quick succession. Short chapters introduce strained coordinates and methods for improving the convergence of series.

This book is clearly designed to draw the reader through a wide range of examples which summarize the wealth of accumulated experience in this area, providing a succinct introduction to the topic at graduate level. For experienced applied mathematicians it gives an exhilarating refresher course, which can excite the most able graduate students. For most graduate students a slower development of topics with some more reassuring, rather than challenging, material would perhaps be useful to build confidence in the subject. The book is based on the author's lectures, and in its brevity reflects the limited time available to a lecturer. However, no important topic is omitted, and it is well suited for people with a background in mechanics who do not wish to become involved in much formal mathematics.

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